6.4 Vectors and Dot Products

warmup: Find a unit vector in the direction of **v** = 6**i** - 2**j**

$u = \frac{v}{||v||} = \frac{6i-2j}{\sqrt{6^2+(-2)^2}} = \frac{6i-2j}{\sqrt{40}} = \frac{6i}{\sqrt{40}} - \frac{2j}{\sqrt{40}}$

$\sqrt{4}\sqrt{10}$

$\frac{6i}{2\sqrt{10}} - \frac{2j}{2\sqrt{10}} = \frac{3i}{\sqrt{10}} - \frac{j}{\sqrt{10}} = \frac{3\sqrt{10}}{10}i - \frac{\sqrt{10}}{10}j$

A vector **u** has magnitude 3 and direction angle 210 degrees. Write the vector in component form & linear combination form.

$u = 3\langle \cos 210^\circ, \sin 210^\circ \rangle$

$= 3\left\langle -\frac{\sqrt{3}}{2}, -\frac{1}{2} \right\rangle$

$\left\langle -\frac{3\sqrt{3}}{2}, -\frac{3}{2} \right\rangle$

$u = 3(\cos 210\, i + \sin 210\, j)$

$3\left(-\frac{\sqrt{3}}{2}i - \frac{1}{2}j\right)$

what is the component form of the vector? $-\frac{3\sqrt{3}}{2}i - \frac{3}{2}j$

What is the linear combination form of the vector?

$-\frac{3\sqrt{3}}{2}i - \frac{3}{2}j$

What we have learned so far...

vector addition **u** + **v**
scalar multiplication 3 **u**

> answers are **vectors**

Now a third operation: Dot Product

You get a real (regular) number NOT a vector!

Dot Product

$u \cdot v$

answer is a scalar (number)

$$u \cdot v = u_1 v_1 + u_2 v_2$$

Example: **u** = <3, 1> and **v** = <5, -6>

Find $u \cdot v$

pull

Examples: Find the dot product:

<5, -4> • <9, -2>

$5(9) + (-4)(-2)$

$45 + 8$

53

	Find the dot product	Find the magnitude
u = $\langle 3, 2 \rangle$	**U•U** <3,2> • <3,2> 9 + 4 13	$\|\|\mathbf{U}\|\|$ $\sqrt{(3)^2+(2)^2}$ $\sqrt{13}$
v = $\langle -5, 3 \rangle$	**V • V** <-5,3> • <-5,3> 25 + 9 34	$\|\|\mathbf{V}\|\|$ $\sqrt{(-5)^2+(3)^2}$ $\sqrt{25+9}$ $\sqrt{34}$
w = $\langle 4, -2 \rangle$	**W • W** <4,-2> • <4,-2> 16 + 4 20	$\|\|\mathbf{W}\|\|$ $\sqrt{(4)^2+(-2)^2}$ $\sqrt{20}$

Conclusion: $u \cdot u = \|u\|^2$

Dot Product and Magnitude

$$u \cdot u = \|u\|^2$$

and

$$\sqrt{u \bullet u} = \|u\|$$

To find the magnitude you can think of using the distance formula OR you can find the dot product and square root that number.

Find the magnitude both ways.

u = <3, 4>

skip

v = <-5,1>

Angle Between Two Vectors

the dot product can help us find the angle between two vectors

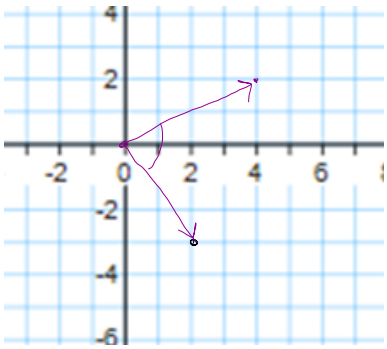
$$\cos\theta = \frac{u \cdot v}{\|u\|\|v\|} \qquad \theta = \cos^{-1}\left(\frac{u \cdot v}{\|u\|\|v\|}\right)$$

Example:

u = 2**i** - 3**j** = <2,- 3>

v = 4**i** + 2**j** = <4, 2>

Find the angle between the two vectors



$\cos^{-1}\left(\frac{8 + -6}{\sqrt{4+9}\cdot\sqrt{16+4}}\right)$

$\cos^{-1}\left(\frac{2}{\sqrt{13}\cdot\sqrt{20}}\right)$

$82.87°$

pull

Orthogonal Vectors

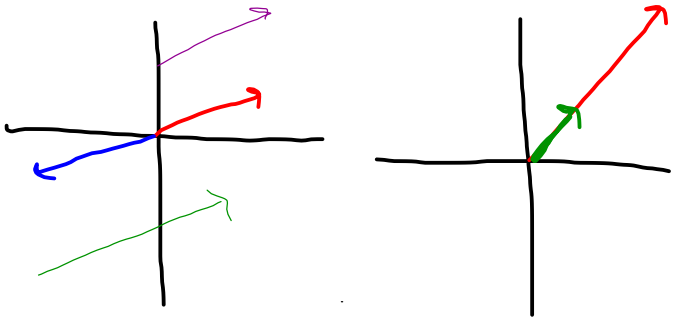
means perpendicular: forms right angle

if dot product = 0 , then the vectors are orthogonal/ perpendicular

Two vectors are parallel if they have the same slope.

Two vectors that are scalar multiples of each other are parallel.

Parallel vectors can touch.



Extra Credit

pg440 #7-37o, 43-49o

Pass out review packet

Quiz tomorrow over vectors.