18. Solve each of the following equations. Write all answers in simplest form.

a) $2\log x = \log 2 + \log(x+12)$

$\log x^2 = \log(2x+24)$

$x^2 = 2x+24$ $\quad x=-4$ extraneous

$x^2-2x-24=0$ $\quad x=6$

$(x+4)(x-6)=0$

b) $\ln x + \ln(x-5) = \ln 6$

$\ln x(x-5) = \ln 6$ $\quad x=6$

$x^2-5x=6$ $\quad x=-1$

$x^2-5x-6=0$ extraneous

$(x-6)(x+1)=0$

c) $\log_2(x+3)+\log_2(x+2)=1$

$\log_2(x+3)(x+2)=1$

$2^1 = x^2+5x+6$ $\quad 0=(x+1)(x+4)$

$0=x^2+5x+4$ $\quad x=-1$ $\quad x=-4$ extraneous

d) $5^{3x-1}=125$

$5^{3x-1}=5^3$

$3x-1=3$

$3x=4$ $\quad x=4/3$

e) $25^{3x-1}=125$

$(5^2)^{3x-1}=5^3$ $\quad 6x=5$

$5^{6x-2}=5^3$ $\quad x=5/6$

$6x-2=3$ $\quad x=$

f) $3\cdot 6^x = 42$

$6^x = 14$

$\log_6 6^x = \log_6 14$

$x=\log_6 14$

if calculator then 1.473

g) $2\cdot e^{7-3x} = -20$

$e^{7-3x} = -10$

$\ln e^{7-3x} = \ln(-10)$

Does not exist

e to a power will never make a negative.

h) $2^{3x-1} = 8^{2x+5}$

$2^{3x-1} = 2^{3(2x+5)}$

$3x-1 = 6x+15$

$-16 = 3x$

$-16/3 = x$

i) $e^{2x}+5e^x-14=0$

$(e^x+7)(e^x-2)=0$

$e^x+7=0$ $\quad e^x-2=0$

$\ln e^x = \ln -7$ $\quad \ln e^x = \ln 2$

$x=\ln(-7)$ $\quad x=\ln 2$

Does NOT exist $\quad x \approx .69315$

19. Evaluate the logarithms without a calculator.

a) $\log_2\sqrt{2}$

$\log_2 2^{1/2}$

$\frac{1}{2}$

b) $\log_5 \sqrt[5]{25}$

$\log_5 25^{1/5}$

$\log_5 (5^2)^{1/5} = \log_5 5^{2/5} = 2/5$

20. Show algebraically that $\log_2\sqrt{8} = \frac{3}{2}$.

$\log_2 8^{1/2} = \frac{3}{2}$

$\log_2 (2^3)^{1/2} = \frac{3}{2}$

$\log_2 2^{3/2} = \frac{3}{2}$

$\frac{3}{2} = \frac{3}{2}$

~~21. Which of the following is equivalent to $\frac{\log 20}{\log 2}$?~~

~~A. log 10~~

~~B. log 18~~

~~C. $\log_2 20$~~ ← correct

~~D. $\log_{20} 2$~~

$\frac{\log_{10} 2^3}{\log_2 16^{1/3}} = \frac{8}{\log_2 (2^4)^{1/3}}$

$= \frac{8}{\log_2 2^{4/3}}$

22. Evaluate the following. (Note: These should be done *without* a calculator.)

a) $e^{2\ln(3)}$

$e^{\ln 3^2}$

$e^{\log_e 3^2} = 3^2 = 9$

b) $\frac{e^{2\ln(3)}}{\ln\sqrt[5]{e}} = \frac{9}{\ln e^{1/5}} = \frac{9}{\frac{1}{5}} = 9\cdot 5 = 45$

c) $\frac{10^{3\log(2)}}{\log_2\sqrt[3]{16}} = \frac{8}{\frac{4}{3}} = 8\cdot\frac{3}{4} = 6$

d) $3^{\log_3 5 - \log_3 7}$

$3^{\log_3 5/7}$

$5/7$

e) $7^{\log_7 6 + \log_7 5}$

$7^{\log_7 6\cdot 5}$

$7^{\log_7 30} = 30$

f) $\frac{\log_5\sqrt{5}+\log_5 1}{\log_2\sqrt[3]{2}} = \frac{\log_5 5^{1/2}+\log_5 1}{\log_2 2^{1/3}}$

$= \frac{\frac{1}{2}+0}{\frac{1}{3}} = \frac{1}{2}\cdot\frac{3}{1} = \frac{3}{2}$